NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 5A : Principles of Mechanics

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VA**

PG-Sc.-AP-17105

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 5A : Principles of Mechanics

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VA**

PG-Sc.-AP-17105

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

(All symbols have their usual meanings)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) If a generalized coordinate is absent in the Lagrangian, will it be also absent in the Hamiltonian ? Justify.

b) Show that the dynamical system for which $2T = r_1 r_2(\dot{r}_1^2 + \dot{r}_2^2)$ and $V = \frac{1}{r_1} + \frac{1}{r_2}$ can be expressed as one of Liouville's type.

c) Show from Lagrange's equations that the orbit of a particle in a central force field lies in a plane.

d) Show that the transformation $Q = -p$, $P = q + \lambda p^2$ (λ, *a* constant) is canonical.

e) If for a certain mechanical system $H = p^2 q^2 - \mu\, pq$, where μ is a real constant, then show that pq is a constant of motion.

f) Show that the quantity $E = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_1} - L$ remains constant during the motion of a closed system.

g) Show that linear momentum is a constant of motion for infinitesimal spatial displacement.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Deduce Lagrange's equation of motion for a conservative and unconnected holonomic system. 7

b) Prove the virial theorem. 3

3. a) Solve the plane pendulum problem using the Hamilton's canonical equations. 5

b) If all the coordinates of a system are cyclic then prove that the coordinates may be found by integration. Also prove that if the system be scleronomic then the coordinates are linear function of time. 5

4. Consider a mechanical system described by he generalized coordinates $q_1, q_2, \ldots, q_n$. Show that the kinetic energy can be formulated as $T = T_2 + T_1 + T_0$, where $T_2 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\dot{q}_1\dot{q}_j$, $T_1 = \sum_{i=1}^{n} b_i \dot{q}_1$, $T_0 = c$, and the quantities a_{ij}, b_i and c are to be determined by you. 10

5. a) In a dynamical system with two degrees of freedom the K.E. and P.E. are given by

$$T = \frac{\dot{q}_1^2}{2(a + bq_2)} + \frac{1}{2}q_2^2\dot{q}_2^2, \; V = c + dq_2,$$

where a, b, c and d are constants. Show that the value of q_2 in terms of time is given by the equation of the form :

$$(q_2 - k)(q_2 + 2k)^2 = h(t - t_0)$$

where h, k and t_0 are constants. 7

b) The Lagrangian of a system with two degrees of freedom is given by

$$L = x\dot{y} + y\dot{x}^2 + \dot{x}\dot{y}$$

obtain the corresponding Hamiltonian. 3

6. Define angle variables. A particle of mass m moves in two dimensions (x, y) having potential $V(x, y) = \frac{1}{2}m w_x^2 x^2 + \frac{1}{2}m w_y^2 y^2$ $(w_x \neq w_y)$. Obtain the action variables expressing the energy in terms of these and hence find the angle variables. 2 + 5 + 3

7. a) Find the equation of the curve which makes the surface area of revolution generated by rotating the curve $y = y(x)$ around the x-axis. 5

b) A particle of unit mass is projected so that its total energy is h in a field of force whose potential is $\phi(r)$ at a distance r from the origin. Show that the differential equation of the path is given by

$$C^2\left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right] = r^4[h - \phi(r)]$$

with C as constant. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 5B : Elements of Continuum Mechanics & Special Theory of Relativity

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

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Enrolment No.

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VB**

PG-Sc.-AP-17106

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

MATHEMATICS

Paper - 5B : Elements of Continuum Mechanics & Special Theory of Relativity

Name (in Block Letter) : ....

Enrolment No.

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VB**

PG-Sc.-AP-17106

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

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এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Define holonomic and non-holonomic system. Give example.

b) Show that for the velocity field given by $v_1 = ax_3 - bx_2$, $v_2 = bx_1 - cx_3$ and $v_3 = cx_2 - ax_1$ the motion is rotational.

c) Find the complex potential of a source.

d) The water in a river moves west at the speed of 6 metre/sec and a boat heads north at 8 metre/sec with respect to the water. Find out the direction and velocity of the boat with respect to the ground.

e) Determine the Cauchy stress quadric at P for a state of stress $(T_{ij}) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{pmatrix}$.

f) Define body and surface force on a continuum body.

g) State Kelvines' theorem on minimum kinetic energy.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Establish the Navier-Stokes equations for incompressible viscous fluid.

b) The stress tensor at P is given by $T_{ij} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$. Determine principal stress and principal directions. 5 + 5

3. State Galilean transformation for two dimensional frame of reference. What was its drawbacks ? What modifications made by Lorentz and derive his transformation. Show that the time interval in the proper frame of clock will be smaller than the observed time interval in the laboratory frame. 2 + 4 + 4

4. a) State the principle of least action. Using it establish the Lagrange's equations of motion.

b) Show that the shortest distance between two points on the surface of a sphere is a great circle. 5 + 5

5. a) Define Poisson bracket. Show that for any three dynamical variables u, v and w the following holds $\{u,\{v,w\}\} + \{v,\{w,u\}\} + w,\{u,v\}\} = 0$, where $\{\cdot\}$ is the Poisson bracket. 5

b) What is canonical transformation ? Show that $(q,p) \to (Q,P)$ defined by $q = \sqrt{\frac{2P}{mw}} \sin Q$, $p = \sqrt{2m\,wP} \cos Q$ is a canonical transformation. 5

6. a) Define circulation (Γ) for any moving fluid. Show that for the conservative force of field air circulation is constant. 5

b) Find the stream function for a two-dimensional source given by the velocity potential $Q = -\frac{m}{2\pi} \log r$, where r is the distance from the source. 5

7. Define dilation, rotation vector, shear, principal strain and the principal axes. For the following displacement relation

$$u_1 = a(X_1 + 2X_2 + 3X_3)$$

$$u_2 = a(-2X_1 + X_2)$$

$$u_3 = a(X_1 + 4X_2 + 2X_3)$$

find all the above defined quantities. 10

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6A : General Topology

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

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To be filled by the Candidate	Serial No. of question answered														**TOTAL**
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Q.P. Code : **PA/4/VIA**

PG-Sc.-AP-17107

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 6A : General Topology

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIA**

PG-Sc.-AP-17107

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

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এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Define basis of a topological space. Find a basis of ($\mathbb{R}$, τ_d), where τ_d is the discrete topology.

b) If $\{\tau_\alpha\}_{\alpha\in\Lambda}$ is a family of topologies on X, show that $\bigcap_{\alpha\in\Lambda} \tau_\alpha$ is also a topology on X.

c) Let Y be a subset $[0,1)\cup\{2\}$ of $\mathbb{R}$ endowed with standard topology. Show that $\{2\}$ is open in the subspace topology on Y.

d) Show that arbitrary intersection of open sets need not be open.

e) In a Hausdorff space if $x \in X$, show that $\bigcap\{\overline{N}_x : N_x \in N_x\} = \{x\}$ where bar denotes the closure and N_x is the nbd. system at x.

f) Show that union of connected subsets need not be connected. State a sufficient condition for the union to be connected.

g) Show that the real line $\mathbb{R}$ with discrete topology is not compact.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Define limit point of a set in a topological space. Find out the limit points of the following subspaces of $\mathbb{R}$, endowed with standard topology.

   i) The set of rational numbers, $\mathbb{Q}$

   ii) $\{1/n : n \in \mathbb{N}\}$

   iii) $\mathbb{R} - \{1, 2, 3, 4\}$. 1 + 2

   b) Write down the definition of a Hausdorff topological space (T_2). Give an example of a topological space which is T_1 but not T_2 with reasons. 1 + 2

   c) Show that (X, τ) is T_2 iff the diagonal $\Delta = \{(x \times x) \mid x \in X\}$ is closed in $X \times X$. 4

3. a) Define continuous function in topological spaces. Let (X, τ_X) and (Y, τ_Y) be two topological spaces and $f : X \to Y$ be a continuous function. Show that for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$, bar denoting the closure. 1 + 2

   b) Show by an example that in a topological space (X, τ) for any two subsets $A, B \subseteq X$,

   i) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ may not be true

   ii) $int(A \cap B) = int(A) \cap int(B)$ holds. 1 + 3

   c) Define a net. Prove that in a topological space (X, τ), a point $u \in X$ is a limit point of $A \subset X$ iff there is a net in $A \setminus \{u\}$ such that the net converges to u. 3

4. a) Define a compact topological space. Prove that every compact subspace of a T_2 space is closed. 1 + 4

   b) Let (X, τ) be compact and (Y, τ') be T_2-topological spaces. If $f : X \to Y$ is a bijective function, then prove that f is a homeomorphism.

   Show that arbitrary union of compact sets need not be compact. 3 + 2

5. a) Define a normal space. Show that a metric space is normal. 3

   b) Let A be a connected subspace of a topological space (X, τ), if $A \subset B \subset \overline{A}$, then show that B is also connected. 4

   c) Prove that continuous image of a connected set is connected. 3

6. a) Prove that a subset of $\mathbb{R}$ with standard topology is connected iff it is an interval. 5

   b) Denine a locally connected space. Prove that (X, τ) is locally connected iff components of each open subspace of X are open in X. 5

7. a) Prove that the one point compactification (X_u, τ_u) of a non-compact topological space (X, τ) is T_2 iff (X, τ) is a locally compact T_2-space. 5

   b) Define a uniform space (X, ν). If τ_ν is the uniform topology on X induced by ν then prove that the τ_ν-closure of $A = \overline{A} = \bigcap \{U(A) : U \in \nu\}$. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Paper - 6B : Functional Analysis

পূর্ণমান : ৫০ — **QUESTION PAPER CUM ANSWER BOOKLET** — মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

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Q.P. Code : **PA/4/VIB**

PG-Sc.-AP-17108

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম (P. G.)

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অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Paper - 6B : Functional Analysis

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIB**

PG-Sc.-AP-17108

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

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Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Notations and symbols have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Show that the closed unit sphere in a normed linear space is convex.

b) Prove or disprove : In a normed linear space the norm function is uniformly continuous.

c) Prove that in an inner product space every orthonormal set is linearly independent.

d) Is l_2 a Hilbert space ? Justify.

e) If A is a self-adjoint operator in a Hilbert space H, then show that A^n is self-adjoint for any natural number n.

f) In an inner product space X, prove that the conditions $< x_n, x > \to < x, x >$ and $\| x_n \| \to \| x \|$ imply $x_n \to x$.

g) Let H be a Hilbert space. Prove that for any bounded linear operator $A : H \to H$, $\left\| A^* A \right\| = \| A \|^2 = \left\| AA^* \right\|$.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) State Hahn Banach theorem in a normed linear space. Prove that for any $x(\neq 0)$ in a normed linear space, $\| x \| = sup\left\{ \frac{|f(x)|}{\|f\|}, f \in X^* \text{ and } f \neq 0 \right\}$.

b) Show that a subspace of a separable metric space is also separable.

c) Show that compactness is not a hereditary property in a metric space. 4 + 4 + 2

3. a) Given that X is a normed linear space and Y is a Banach space. Show that the space $B(X,Y)$ of all bounded linear operators from X into Y is a Banach space.

b) Given that the conjugate space X^* of a normed linear space X is separable. Show that X is separable. 5 + 5

4. a) State and prove Riesz Representation theorem in a Hilbert space.

b) If X is a normed linear space such that the unit sphere $\{x \in X : \| x \| = 1\}$ is compact, then prove that X is finite dimensional. 5 + 5

5. a) Let M be a proper closed subspace of a normed linear space X and let $x_0 \in X \setminus M$. If $d = \inf_{x \in M} \| x_0 - x \|$ then show that there exists an $f \in X^*$ such that $\| f \| = 1$, $f(x_0) = d$ and $f(x) = 0$ for all $x \in M$.

b) Let $L_2[0,2\pi]$ be the real Hilbert space of all square integrable functions f over $[0,2\pi]$ with inner product given by $(f,g) = \int_0^{2\pi} fg \, dt$, $f,g \in L_2[0,2\pi]$.

Show that $e_0(t) = \frac{1}{\sqrt{2\pi}}$, $e_n(t) = \frac{\cos nt}{\sqrt{\pi}}$, $n = 1,2,\ldots$

where $0 \leq t \leq 2\pi$ forms an orthonormal sequence in $L_2[0,2\pi]$. 5 + 5

6. a) State and prove Bessel's inequality.

b) Show that the collection of all self-adjoint operators forms a closed real linear subspace of the space of all continuous linear operators that map H into itself, where H is a Hilbert space.

c) Show that the space $c[a,b]$ with sup norm is not a Hilbert space. 4 + 4 + 2

7. a) Suppose that $A : H \to H$ is a self-adjoint operator where H is a Hilbert space. Show that $\| A \| = sup\{ < Ax, x > : \| x \| = 1\}$.

b) Prove that in a separable Hilbert space H, every orthonormal system is countable.

c) For two vectors x and y in an inner product space X, prove that $x \perp y$ if and only if $\| y \| \leq \| \alpha x + y \|$ for any scalar α. 5 + 3 + 2

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 7A : Differential Equations and Integral Transformations

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

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Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VIIA**

PG-Sc.-AP-17109

Signature of Evaluator with Date

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 7A : Differential Equations and Integral Transformations

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIIA**

PG-Sc.-AP-17109

Received Answer Booklet
Signature with seal by the Study-Centre

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Do Not Write Anything Here

(Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Show that $L[\cos \omega t] = \dfrac{p}{p^2 + \omega^2}$.

b) State and prove the shifting theorem of Laplace transform.

c) If $F(p)$ is the Laplace transform of a function $f(t)$, which is piecewise continuous in any finite interval of t and is of exponential order $O(e^{at})$ at $t \to \infty$, then show that $\lim\limits_{p \to \infty} F(p) = 0$.

d) How Fourier transform of a function $f(x)$ of real variable x is defined ?

e) By the use of Fourier transform reduce the differential equation,

$$6\frac{d^2u}{dx^2} + 4\frac{du}{dx} + 4 = 0$$

with the given initial conditions,

$$\left(\frac{du}{dx}\right)_0 = 0,\ (u)_0 = 1$$

to an algebraic equation.

f) State the convolution theorem of Laplace transform.

g) Write the expression for the convolution of the two functions $f(x)$ and $g(x)$ and then state the convolution theorem.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Considering the functions $f_1(x)$ and $f_2(x)$ defined by

$$\left.\begin{aligned} f_1(x) &= 1, |x| \le a \\ &= 0, |x| > a \end{aligned}\right\} \quad \begin{aligned} f_2(x) &= 1, |x| \le b \\ &= 0, |x| > b \end{aligned}$$

and using Parseval's relation, show that

$$\int_0^\infty \frac{\sin ax \sin bx}{x^2}\, \mathrm{d}x = \frac{\pi a}{2},\ 0 < a < b.$$ 10

3. Find the Fourier inversion of $|k|^{1/2}\, \overline{f}(k)$, where $\overline{f}(k)$ is the Fourier transform of $f(x)$. 10

4. If $L[f(t)] = F(p)$ in the domain $Re(p) > a$ and $\phi(t) = \int_0^t f(\tau)\, \mathrm{d}\tau$, then show that $L[\phi(t)] = \frac{1}{p} F(p)$, which exists in the domain $Re(p) > a$. Here L denotes the Laplace transform. 10

5. Find $L^{-1}\left[\frac{p}{(p^2 + a^2)^2}\right]$ by the use of convolution theorem, where L denotes Laplace transform. 10

6. Use Laplace transform to find the solution of the equation $\frac{\mathrm{d}^3 x}{\mathrm{d}t^3} + \frac{\mathrm{d}x}{\mathrm{d}t} = \sin t$, satisfying the initial conditions, $x(0) = 0$, $x'(0) = -2$, $x''(0) = 0$. 10

7. By the use of the function $f(x)$ defined by

$f(x) = 1 - |x|$, when $|x| \le 1$

$=$ 0, when $|x| > 1$

and using Fourier inversion theorem show that $\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} \mathrm{d}x = \pi$. 10

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) : জুন, ২০২০ (June, 2020)**

MATHEMATICS

Paper - 7B : Integral Equations and Generalised Functions

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

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Enrolment No.

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered													TOTAL
For Evaluator's only	Marks awarded													

Q.P. Code : **PA/4/VIIB**

PG-Sc.-AP-17110

Signature of Evaluator with Date

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

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অনুশীলন পত্র **(Assignment) : জুন, ২০২০ (June, 2020)**

MATHEMATICS

Paper - 7B : Integral Equations and Generalised Functions

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Enrolment No.

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIIB**

PG-Sc.-AP-17110

Received Answer Booklet
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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Use Laplace transform to solve $\int_0^x \sqrt{x-t}\ \phi(t)\mathrm{d}t = x^{5/2}, x > 0$.

b) Show that the integral equation

$$\phi(x) - \int_0^1 (5x^2 - 3)t^2\phi(t)\mathrm{d}t = 0,\ 0 < x < 1,$$

has only trivial solution.

c) Reduce the integral equation

$$\phi(x) - \int_0^x \phi(t)\mathrm{d}t = x,\ \ x > 0$$

to an ordinary differential equation with appropriate initial condition.

d) Use the method of successive approximations to solve $\phi(x) + \int_0^x \phi(t)\mathrm{d}t = 1,\ x > 0$.

e) Show that $y(x) = \cos 2x$ is a solution of the equation

$$y(x) = \cos x + 3\int_0^{\pi} k(x,t)y(t)\mathrm{d}t,\ 0 < x < \pi$$

where $k(x,t) = \begin{cases} \sin x \cos t, & 0 \le x < t \\ \cos x \sin t, & t < x \le \pi \end{cases}$

f) Obtain an integral equation corresponding to following initial value problem

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = \cos x,\ 0 < x < 1,\ y(0) = y'(0) = y_0.$$

g) Find the first three iterated Kernels of $\phi(x) = 1 + \int_0^x e^{x-t}\phi(t)\mathrm{d}t,\ 0 < x < 1$.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Solve the following Fredholm integral equation for all values of the parameter λ

$$\phi(x) = 1 + \lambda \int_0^1 (1 - 3xt)\phi(t)\,dt,\ 0 < x < 1.$$ 10

3. a) Use the method of successive approximations to solve

$$u(x) = 2 - \int_0^x (x - t)u(t)\,dt,\ 0 < x < 1.$$

b) Use Laplace transform technique to solve

$$\int_0^x \frac{\phi(t)}{\sqrt{x - t}}\,dt = f(x),\ x > 0,\ f(0) = 0.$$ 10

4. Reduce the following integral equation to a boundary value problem.

$$\phi(x) - \lambda \int_0^x t(1 - x)\phi(t)\,dt - \lambda \int_x^1 x(1 - t)\phi(t)\,dt = x,\ 0 < x < 1.$$

Use Hilbert-Schmidt theorem to solve the above integral equation. 4 + 6

5. a) Convert the following intial value problem to an integral equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,\ 0 < x < 1,$$

$$y(0) = 1,\ y'(0) = 0.$$

b) Derive an equivalent Fredholm integral equation for the following boundary value problem :

$$\frac{d^2y}{dx^2} + y = x,\ y(0) = 1,\ y(\pi) = \pi - 1,\ 0 < x < \pi.$$ 5 + 5

6. a) Find the first and second iterated kernel of

$$\phi(x) - \lambda \int_0^{\pi/2} \sin(x - t)\phi(t)\,dt = f(x),\ 0 < x < \frac{\pi}{2}.$$

b) Find the resolvent kernel for the integral equation

$$g(s) = f(s) + \lambda \int_{-1}^1 (st + s^2t^2)g(t)\,dt,\ -1 < s < 1.$$ 5 + 5

7. a) Show that the integral equation

$$\phi(x) - \frac{1}{\pi}\int_0^{2\pi} \sin(x + t)\phi(t)\,dt = f(x),\ 0 < x < 2\pi$$

possesses no solution for $f(x) = x$ but possesses infinite solution when $f(x) = 1$.

b) Find the boundary value problem that is equivalent to the integral equation

$$y(x) - \lambda \int_{-1}^1 (1 - |x - t|)y(t)\,dt = 0,\ -1 < x < 1.$$ 5 + 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 8A : Differential Geometry

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

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Q.P. Code : **PA/4/VIIIA**

PG-Sc.-AP-17111

Signature of Evaluator with Date

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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Paper - 8A : Differential Geometry

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIIIA**

PG-Sc.-AP-17111

Received Answer Booklet

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5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

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(Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Define a scalar in the sense of tensor.

b) If A_{mn} is a skew-symmetric tensor and B^i is a contravariant vector, is $A_{mn}B^m B^n = 0$?

c) Define fundamental metric tensor.

d) What do you mean by a contravariant vector of length l ?

e) Calculate the curvature of a straight line.

f) Define a developable surface.

g) When is a surface called minimal ?

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) If f is a scalar function of co-ordinates (x^i), then show that dx^i is a contravariant vector and $\frac{\partial f}{\partial x^i}$ is a covariant vector. 5

   b) Prove that the gradient of a function is a covariant vector. 5

3. a) Prove that the inner product of two tensors A_q^p and B_m^{ij} is a tensor of type (2, 1). 5

   b) Show that conjugate symmetric tensor are the components of a symmetric contravariant tensor of type (2, 0). 5

4. a) Evaluate the Christoffel symbols of both kinds for spaces where $g_{ij} = 0$, if $i \neq j$. 5

   b) Define scalar curvature and hence show that in an Einstein space, it is constant provided dimension of the space is greater than 2. 5

5. a) If the intrinsic derivative of a vector A along a curve C vanishes at all points of C, show that the magnitude of A is constant along C. 5

   b) Define a helix. Show that the ratio of curvature and torsion of such curve is constant. 5

6. a) Find an expression for the angle between two intersecting curves on a surface. 5

   b) Define a developable surface. Explain with an example. 5

7. a) Show that a surface is a sphere if and only if the second fundamental form is a non-zero constant multiple of its fundamental form. 5

   b) Find a relation between three fundamental forms of the surface. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Paper - 8B : Graph Theory

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														**TOTAL**
For Evaluator's only	Marks awarded														

Q.P. Code : **PA/4/VIIIB**

PG-Sc.-AP-17112

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম (P. G.)

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অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Paper - 8B : Graph Theory

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **PA/4/VIIIB**

PG-Sc.-AP-17112

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
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5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

Do Not Write Anything Here

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Draw a graph having degree sequence (1, 2, 2, 4, 5).

b) Let $A = \{ a, b, c, d \}$ and ρ be a relation on A defined by

$\rho = \{(a,a),(b,a),(a,b),(c,a),(a,c),(b,b),(c,c),(d,d),(b,d),(d,b)\}$.

Draw the digraph representing ρ. Is it an equivalence relation ?

c) A tree can never be a regular graph — True or False ? Justify your answer.

d) Define incidence matrix with an example.

e) Show that if the degree of each vertex of a graph G is at least 2, then G contains a circuit.

f) If G is a forest with n vertices & k components, find the number of edges of G.

g) Differentiate between (i) Trail & Path, (ii) Circuit & Cycle.

First Answer :

Second Answer :

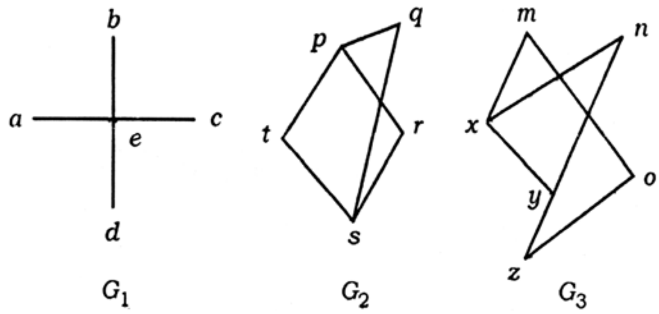
Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Establish Euler's formula for a connected planar graph. 5

b) Which of the following graphs are bipartite ? Explain your answer with justification. 5



3. a) Define a Directed graph. Briefly describe the process of representation of binary relations on finite sets by directed graphs with examples. 7

b) What do you understand by 'shortest spanning tree' of a graph ? 3

4. a) Let p denote the number of vertices of a tree T of degree 1 and q denote the number of vertices of degree ≥ 3. If T contains at least two vertices, prove that $p \geq q+2$. Also find the condition for equality. 5

b) A cycle on n vertices is isomorphic to its complement. Find the value of n. 5

5. a) What is a rooted tree and what is level of its vertics ? Define an m-tree. Prove that an m-tree has at most m_p vertices at level p. 5

b) Prove that in a connected planar graph, in which every vertex is of at least degree 3, there exists a region with fewer than 6 edges in the boundary. 5

6. a) Define isomorphic graphs. Prove that any two simple connected graphs with n vertices, all of degree 2, are isomorphic. 4

b) Prove that a connected graph G has a Euler trail if and only if it has exactly two odd-degree vertices. 6

7. a) Let G be a simple connected graph with 11 or more vertices. Show that either G or $\overline{G}$ is non-planar. 6

b) Describe Kruskal's algorithm for finding a shortest spanning tree of a connected weighted graph with an example. 4

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9A(i) : Advanced Complex Analysis & Paper - 9A(ii) : Operations Research

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

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Enrolment No.												

Study Centre Name : .... Code : ....

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For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/IXA(i))/(PA/4/IXA(ii))**

PG-Sc.-AP-17113

Signature of Evaluator with Date

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স্নাতকোত্তর পাঠক্রম (P. G.)

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অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9A(i) : Advanced Complex Analysis & Paper - 9A(ii) : Operations Research

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **(PA/4/IXA(i))/(PA/4/IXA(ii))**

PG-Sc.-AP-17113

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

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PG-Sc.-AP-17113

Special Paper : Pure Mathematics

Paper - 9A(i) : Advanced Complex Analysis

(Symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define the exponent of convergence of zeros of an entire function.

   b) Find the order of $e^{z/2}$.

   c) Show that for fixed R and ϕ, Poisson's Kernel $\dfrac{R^2 - |z|^2}{|Re^{i\phi} - z|^2}$ is harmonic in $|z| < R$.

   d) State Mittag-Leffler theorem.

   e) State Riemann mapping theorem.

   f) State Schwarz Reflection principle.

   g) Define an entire function. Give an example of it.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Prove that any harmonic function defined on a domain D has the mean value property in D. 4

   b) Let f be a function regular in the closed disc $|z| \le R$ and let $v(r,\theta)$ be its imaginary part. If $v(r,\theta) \ge 0$, then prove that $\frac{R-r}{R+r}v(0,0) \le v(r,\theta) \le \frac{R+r}{R-r}v(0,0)$, where $0 \le r < R$. 6

3. a) Let $u(x,y) \not\equiv$ constant be harmonic in a domain D. Then prove that $u(x,y)$ has neither a maximum nor a minimum at any interior point of D. 4

   b) Let $f(z)$ be an analytic function in a domain D containing z_0. If $f'(z_0) \neq 0$ then $f(z)$ is conformal at z_0. 6

4. a) State and prove Schwarz lemma. 6

   b) Test for convergence of the infinite product $\prod_{n=0}^{\infty}\left(1-\frac{z^2}{n^2}\right)$. 4

5. a) If $f(z)$ is an entire function and does not vanish on $\mathbb{C}$, then show that $f(z)$ is of the form $f(z) = e^{g(z)}$, where $g(z)$ is an entire function. 4

   b) State and prove Poisson-Jensen formula. 6

6. a) Let $f(z)$ be a non-constant analytic function in $|z| < R$. Then show that $M(r)$ is a strictly function of r in $0 \le r < R$. 4

   b) State and prove Jensen's inequality. 6

7. a) If $f(z)$ be an entire function with finite order ρ, then $n(r) = O(r^{\rho+\epsilon})$ for $\epsilon > 0$ and for sufficiently large r. 5

   b) Prove that if ρ is not an integer then $\rho = \rho_1$. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

PG-Sc.-AP-17113

Special Paper : Applied Mathematics

Paper - 9A(ii) : Operations Research

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Discuss the post-optimality effect when a variable is deleted from an LPP.

b) Write down the Kuhn-Tucker conditions for the following problem :

Maximize $f(x_1, x_2, x_3) = -x_1^2 - x_2^2 - 2x_3^2 + 5x_1 + 3x_2$

subject to $x_1 + 2x_2 \le 3$, $2x_1 + 3x_2 \le 12$.

c) What are golden section, golden ratio and golden number ?

d) What is the utility of mixed integer programming technique ?

e) Write short notes on steepest descend method for solving non-linear unconstrained optimization problem.

f) Under what conditions the Kuhn-Tucker necessary conditions are sufficient for maximization and minimization problems with ≤ type constraints.

g) What are the differences between regular simplex method and Wolfe's modified simplex method ?

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Find the extreme points of the function

$f(x_1, x_2) = 2x_1^3 + 3x_2^3 + 5x_1^2 + 4x_2^2 + 6$

b) Find the dimension of a cylindrical tin (with top and bottom) made up of sheet metal to maximize its volume such that the total surface area is equal to 24π. 4 + 6

3. a) Use revised simplex method to solve the following LPP :

Maximize $Z = x_1 + 2x_2$

subject to $x_1 + x_2 \le 3$, $x_1 + 2x_2 \le 5$, $3x_1 + x_2 \le 6$, $x_1, x_2 \ge 0$.

b) Given the LPP

Maximize $Z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \le 18$, $x_1 \le 4$, $x_2 \le 6$ and x_1, $x_2 \ge 0$.

Discuss the effect on the optimality of the solution when the objective function is changed to $3x_1 + x_2$. 5 + 5

4. a) Discuss the criteria for selection of incoming and outgoing vectors in dual simplex method.

b) Using artificial constraint method, solve the following problem by dual simplex method and show that the problem has no feasible solution.

Maximize $Z = -x_1 + x_2$

subject to $x_1 - 4x_2 \ge 5$, $x_1 - 3x_2 \le 1$, $2x_1 - 5x_2 \ge 1$, $x_1, x_2 \ge 0$. 4 + 6

5. a) Write down the algorithm of cutting plane method for solving non-linear constrained optimization problem.

b) Using cutting plane method, solve the following problem :

Maximize $f(x_1, x_2) = 1 - 4x_1 - 2x_2$

subject to $2(x_1 - 2)^2 + (x_2 - 3)^2 \ge 12$, $2x_1 + x_2 \le 3$, $0 \le x_1, x_2 \le 5$, (use $\varepsilon = 0\cdot 2$).

3 + 7

6. a) Minimize $f(x) = \begin{cases} 2\sqrt{x}, & x \le 1 \\ 3 - x, & x > 1 \end{cases}$

in the interval [0, 5] by golden section method up to six experiments.

b) Using Davidon-Fletcher-Powell (DFP) method minimize

$f(x_1, x_2) = 8x_1^2 + 4x_2^2 - 24x_1 + 16x_2 + 35$ with $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$ as the starting point. 4 + 6

7. a) Solve the following all integer programming problem :

Maximize $Z = 3x_1 + 4x_2$

subject to $3x_1 + 2x_2 \le 8$

$x_1 + 4x_2 \ge 10$

$x_1, x_2 \ge 0$ and are integers.

b) Applying Wolfe's method, solve the following quadratic programming problem :

Maximize $Z = 2x_1 + x_2 - x_1^2$

subject to $2x_1 + 3x_2 \le 6$, $2x_1 + x_2 \le 4$ and $x_1, x_2 \ge 0$. 4 + 6

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9B(i) : Advanced Topology & Paper - 9B(ii) : Mathematical Models In Ecology

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

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Study Centre Name : .... Code : ....

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Q.P. Code : **(PA/4/IXB(i))/(PA/4/IXB(ii))**

PG-Sc.-AP-17114

Signature of Evaluator with Date

NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **(P. G.)**

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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 9B(i) : Advanced Topology & Paper - 9B(ii) : Mathematical Models In Ecology

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **(PA/4/IXB(i))/(PA/4/IXB(ii))**

PG-Sc.-AP-17114

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / **Important Instruction**

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এখানে কিছু লিখবেন না

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Special Paper : Pure Mathematics

Paper - 9B(i) : Advanced Topology

(Symbols and Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* of the following : $2 \times 5 = 10$

 a) Show that the space of rationals with the induced topology from the usual topology of reals is not locally compact.

 b) Prove that the continuous image of a sequentially compact set is sequentially compact.

 c) Give an example of a paracompact topological space which is not compact.

 d) For two compactifications (f, Y) and (g, Z) if $(f, Y) \le (g, Z)$ and $(g, Z) \le (f, Y)$ then show that (f, Y) and (g, Z) are equivalent.

 e) Is the real number space endowed with the cofinite topology metrizable ? Answer with reasons.

 f) In a metric space (X, d) prove that $x \in \overline{A}$ iff $d(x, A) = 0$ where $x \in X$, $A \subset X$.

 g) Show that any metric space is a uniform space.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Show that a filter $\Im^*$ is an ultrafilter in X iff any subset A of X which intersects every member of $\Im^*$ belongs to $\Im^*$. 5

   b) For a topological space (X, τ), prove that following are equivalent :

   i) X is compact

   ii) Every net in X has a convergent subnet

   iii) Every maximal net in X converges in X. 5

3. a) Define countable compactness. Show that (X, τ) is countably compact iff every infinite set $S \subset X$ has an ω-accumulation point in X. 5

   b) Let X be a Tychonoff space. Prove that X is locally compact iff the remainder $\beta X \setminus X$ is closed. 5

4. a) State and prove Stone-Cech theorem. 6

   b) Prove that every Hausdorff para-compact space is regular. 4

5. a) If every open cover of a topological space X has a closed locally finite refinement then show that X is para-compact. 6

   b) Let τ_l be the lower limit topology on R. Assuming that $(\mathbb{R}, \tau_l)$ is Lindeloff, prove that $(\mathbb{R}, \tau_l)$ is a paracompact space. 4

6. a) Prove that $M \subset c[a,b]$ is relatively compact iff it is uniformly bounded and equicontinuous. 6

   b) Show that the derived set of a countably compact set in a metric space is countably compact. 4

7. a) Prove that every uniformizable space is completely regular. 5

   b) Prove that the filter associated with a Cauchy net is a Cauchy filter and conversely every net associated with a Cauchy filter is Cauchy. 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - 9B(ii) : Mathematical Models In Ecology

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Define an ecosystem. Describe the different components of ecosystem.

   b) Define a dynamical model. What do you mean by a deterministic model ?

   c) Write down the Malthus model of population growth. What are the drawbacks of the model ?

   d) For the differential equation $\frac{dx}{dt} = f(x)$, define stability and asymptotic stability of an equilibrium point x^* in the sense of Lyapunov.

   e) Define a fixed point x^* of the difference equation $x_{n+1} = f(x_n)$. Write down a condition for asymptotic stability of a fixed point x^* of this equation.

   f) Explain with an example the concept of time-delay in the mathematical modelling of a population growth.

   g) Write down the Lotka-Volterra predator-prey model.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Write down the logistic model equation of population growth and explain the different terms involved in the equation. Hence explain the concepts of carrying capacity and intra-species competition. Investigate the asymptotic stability of the equilibrium points of the model. Solve the logistic equation for a given initial condition and discuss the behaviour of the solution if time approaches infinity. 10

3. a) Describe "Cobweb diagram" method of solution of the difference equation $x_{n+1} = f(x_n)$. 5

   b) The growth of a population satisfies the difference equation $x_{n+1} = \dfrac{kx_n}{b + x_n}$, $b, k > 0$.

   When does the positive fixed point of the equation exist ? Discuss stability of the positive fixed point, when it exists. 5

4. a) Solve the second order difference equation $x_{n+1} = x_n + x_{n-1}$ with $x_0 = 0$ and $x_1 = 1$. What is Golden mean ? 6

   b) Find the equilibrium points of the Lotka-Volterra predator-pray model. Discuss their stability. 4

5. State and prove Bendixson's negative criterion for the existence of a closed orbit contained in a domain $D \subset \mathbb{R}^2$. 10

6. Find the equilibrium points of the following predator-prey system :

$$\frac{dx}{dt} = x\left(1 - \frac{x}{30}\right) - \frac{xy}{x+10},$$

$$\frac{dy}{dt} = y\left(\frac{x}{x+10} - \frac{1}{3}\right).$$

Investigate their stability. 10

7. a) For the Lotka-Volterra two-species competition model, show that there is no closed orbit in the interior of the first quadrant. 3

   b) Define a cooperative system. Prove that the orbit of a cooperative system either converges to equilibrium or diverges to infinity. 7

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10A(i) : Advanced Differential Geometry & Paper - 10A(ii) : Fluid Mechanics

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.

Name (in Block Letter) : ....

Enrolment No.

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/XA(i))/(PA/4/XA(ii))**

PG-Sc.-AP-17115

Signature of Evaluator with Date

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স্নাতকোত্তর পাঠক্রম **(P. G.)**

STUDENT'S COPY

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০ **(June, 2020)**

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10A(i) : Advanced Differential Geometry & Paper - 10A(ii) : Fluid Mechanics

Name (in Block Letter) : ....

Enrolment No.

Study Centre Name : .... Code : ....

Q.P. Code : **(PA/4/XA(i))/(PA/4/XA(ii))**

PG-Sc.-AP-17115

Received Answer Booklet
Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.

Detail schedule for submission of assignment for the PG Term End Examination June, 2020

1.	Date of Publication	: 20/06/2020
2.	Last date of Submission of answer script by the student to the study centre	: 19/07/2020
3.	Last date of Submission of marks by the examiner to the study centre	: 16/08/2020
4.	Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.)	: 23/08/2020
5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

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Special Paper : Pure Mathematics
Paper - 10A(i) : Advanced Differential Geometry

(Notations have their usual meanings.)

A manifold always means a differential manifold of class c^{∞}.
Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) When is a function said to be a diffeomorphism ?

   b) Define a differentiable curve on a manifold.

   c) Using $[X, X] = \theta$, show that $[X, Y] = -[Y, X]$.

   d) When is a vector field said to be invariant under a smooth function ?

   e) Define a 1-form on a manifold.

   f) Show that $L_a L_b \neq L_b L_a$ in general.

   g) When is a linear connection said to be symmetric ? Calculate $[X, Y]$ in such case.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Define a differentiable manifold of class c^{∞}. 3

b) Show that S^1 is an one-dimensional differentiable manifold. 7

3. a) Define a tangent vector on a manifold. 3

b) If $X = \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} - x^3 \frac{\partial}{\partial x^3}$, $Y = x^2 \frac{\partial}{\partial x^2}$, $f = x^1 x^2 + \sin^2 x^3$

i) compute Xf

ii) compute $[X, Y]$. 4 + 3

4. a) Show that a necessary and sufficient condition for two vector fields *X, Y* respectively on two manifolds *M, N* to be *f*-related, is that

$f^*((f_* X)g) = X(f^* g)$

where $f : M \to N$ is a c^{∞} map and $g \in c^{\infty}(f(p))$, $p \in M$. 6

b) Does $X = -e^{x^1} \frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^2}$ generates one parameter group of transformations ? Justify your answer. 4

5. a) Define the total differential of a differentiable function on a manifold *M*. Show that it is a 1-form of *M*. 2 + 2

b) For any 0-form ω, prove that $d(f^* \omega) = f^*(d\omega)$. 6

6. a) Define a linear connection in the sense of Koszul. 3

b) If $X = \xi^i \frac{\partial}{\partial x^i}$ and $Y = \eta^j \frac{\partial}{\partial x^j}$, find $\nabla_X^{\ Y}$ and $\nabla_Y^{\ X}$. 4 + 3

7. a) When is a Riemannian manifold said to be of constant curvature ? Define an Einstein manifold and show that every Riemannian manifold of constant curvaure is an Einstein manifold. 2 + 2 + 4

b) Define a semi-symmetric metric connection. 2

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Special Paper : Applied Mathematics

Paper - 10A(ii) : Fluid Mechanics

(Notations / symbols have their usual meaning.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) Show that for a two-dimensional irrotational motion velocity potential and stream function satisfy Laplace's equation.

   b) Find the velocity components for a liquid motion represented by the complex potential $\omega = A \log z$. Draw the stream lines.

   c) State Blasius theorem for the pressure thrust and its momentum on a cylindrical body in an incompressible homogeneous liquid.

   d) What is the vorticity vector in a fluid motion ? Define irrotational motion and vortex motion. Write down equation of vortex lines.

   e) Write down the constitutive relation for a viscous fluid. Then write down the Navier-Stokes equation for the flow of a viscous fluid.

   f) Define source, sink and doublets in a liquid motion.

   g) Show that for a vortex motion, strength of a vortex tube is constant along the length for all times.

   h) Explain standing waves and progressive waves along the surface of a water body.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Deduce the equation of motion of a sphere moving in an incompressible ideal fluid at rest at infinity with velocity U along the axis of 'Z'. Hence show that the effect of the presence of the liquid is to reduce the external force in the ratio $(\sigma-\rho):\left(\sigma+\frac{1}{2}\rho\right)$, σ and ρ being the densities of the sphere and the liquid respectively.

b) A sphere of radius 'a' is made to move in an incompressible perfect fluid with non-uniform velocity 'u' along the x-axis. If the pressure at infinity is zero, prove that at a point 'x' in advance of the centre, pressure 'p' is given by the following equation :

$$p=\frac{1}{2}\rho a^3\left\{\frac{\dot{u}}{x^2}+u^2\left(\frac{2}{x^3}-\frac{a^3}{x^6}\right)\right\}.$$ 5 + 5

3. a) Show that if there is a streaming past a fixed circular cylinder with uniform velocity U in the negative direction of X-axis and there is a circulation of strength k, then the cylinder experiences an upward lift amounting $\rho k U$, ρ being the density of the liquid.

b) State and prove Milne-Thomson circle theorem. Apply the theorem to find the complex potential of a uniform stream at incidence β with positive direction of X axis. 5 + 5

4. a) Show that a progressive wave in a finite depth of liquid with a free surface, liquid particle describes an ellipse about its mean position near the free surface.

b) Discuss the motion of capillary waves in a channel of uniform depth. 5 + 5

5. Describe Karman Vortex street. Discuss the motion of rectilinear vortices lying on such a street. Find out the velocity of lower row and show that it is equal to $\frac{k}{2a}\tan h\frac{\pi b}{a}$, where vortices '$k$' in the upper row are at the points $ma+\frac{i}{2}b$ and vortices '$-k$' in the lower row at $\left(m+\frac{1}{2}\right)a-\frac{1}{2}ib$, $m=0, \pm 1, \pm 2$. 5 + 5

6. a) Consider a steady flow of an incompressible viscous fluid through a pipe with rectangular cross-section bounded by the plane $x=a$, $x=-a$, $y=b$, $y=-b$. Find the volume rate of flow at any cross-section.

b) Define axisymmetric motion and Stokes stream function. Obtain the differential equation satisfied by Stokes' stream function and velocity potential for an irrotational motion symmetric about an axis. 5 + 5

7. a) Calculate the energy which is dissipated in a viscous liquid in motion due to internal friction.

b) Obtain boundary layer approximation of Navier-Stokes equation for a flow of viscous incompressible liquid passing over a flat plate and discuss Blasius solution for boundary layer over a flat plate. 5 + 5

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

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স্নাতকোত্তর পাঠক্রম (P. G.)

অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10B(i) : Advanced Functional Analysis & Paper - XB(ii) : Mechanics of Solids

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।

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Enrolment No.												

Study Centre Name : .... Code : ....

To be filled by the Candidate	Serial No. of question answered														TOTAL
For Evaluator's only	Marks awarded														

Q.P. Code : **(PA/4/XB(i))/(PA/4/XB(ii))**

PG-Sc.-AP-17116

Signature of Evaluator with Date

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স্নাতকোত্তর পাঠক্রম (P. G.)

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অনুশীলন পত্র (Assignment) : জুন, ২০২০ (June, 2020)

MATHEMATICS

Special Paper : Pure Mathematics & Applied Mathematics

Paper - 10B(i) : Advanced Functional Analysis & Paper - XB(ii) : Mechanics of Solids

Name (in Block Letter) : ....

Enrolment No.												

Study Centre Name : .... Code : ....

Q.P. Code : **(PA/4/XB(i))/(PA/4/XB(ii))**

PG-Sc.-AP-17116

Received Answer Booklet

Signature with seal by the Study-Centre

জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

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5.	Last date of submission of marks by the study centre to the Department of C.O.E. on or before	: 31/08/2020

এখানে কিছু লিখবেন না

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Special Paper : Pure Mathematics

Paper - 10B(i) : Advanced Functional Analysis

(Notations and symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

 a) Give an example with proper justification of a symmetric set in a vector space X, which is not balanced.

 b) Let G be an open set in a topological vector space X and $A \subset X$. Prove that $A + G$ is open in X.

 c) When is a normed linear space said to be strictly convex ? Give an example of it.

 d) Give an example with proper justification of a linear operator which is bounded but not compact.

 e) Let H be a complex Hilbert space and $T : H \to H$ be a bounded self-adjoint linear operator. Prove that all the eigenvalues of T (if they exist) are real.

 f) Let X be a complex Banach algebra with identity e. If $x \in X$ and there are $y, z \in X$ such that $yx = e$ and $xz = e$, then show that x is invertible and $y = z = x^{-1}$.

 g) Let X be a complex Banach algebra with identity e and $x \in X$ be such that $\|x\| < 1$. Show that $\left\|(e-x)^{-1} - e - x\right\| \leq \dfrac{\|x\|^2}{1 - \|x\|}$.

 h) Let P_1, P_2 be two orthogonal projection operators of a Hilbert space H onto the closed subspaces Y_1, Y_2 of H respectively such that $P_1P_2 = P_2P_1$. Show that P_1P_2 is also an orthogonal projection operator on H.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. a) Prove that in a topological vector space X, the following statements are equivalent :

   i) A subset E of X is bounded.

   ii) If $\{x_n\}_{n \in \mathbb{N}}$ is a sequence in E and $\{\alpha_n\}_{n \in \mathbb{N}}$ is a sequence of scalars such that $\lim_{n \to \infty} \alpha_n = 0$, then $\lim_{n \to \infty} \alpha_n x_n = \underline{0}$ in X. 3 + 3

   b) Prove that every locally compact topological vector space is finite dimensional. 4

3. Let X be a topological vector space over $\mathbb{R}$ and f be a linear functional on X such that $f(x) \neq 0$ for some $x \in X$. Prove that the following statements are equivalent :

   i) f is continuous

   ii) The null space $N(f)$ of f is closed

   iii) $N(f)$ is not dense in X

   iv) f is bounded in some neighbourhood of $\underline{0}$ in X. 10

4. Let X be a vector space over $\mathbb{R}$, A be a convex and absorbing subset of X and p_A be the Minkowski functional of A. Then prove the following :

   i) $p_A(x+y) \leq p_A(x) + p_A(y)$, $\forall\, x, y \in X$

   ii) $p_A(tx) = t\, p_A(x)$ for all scalars $t \geq 0$ and for all $x \in X$

   iii) If A is balanced, then p_A is a seminorm

   iv) If $B = \{x \in X : p_A(x) < 1\}$ and $C = \{x \in X : p_A(x) \leq 1\}$, then $B \subset A \subset C$ and $p_A = p_B = p_C$. 10

5. a) Prove that a topological vector space X is normable if and only if there is a convex bounded neighbourhood of $\underline{0}$ in X. 5

   b) Prove that the dual space of l_p is isomorphic to the sequence space l_q, where $1 < p, q < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. 5

6. a) Prove that a bounded linear operator $P : H \to H$ on a Hilbert space H is an orthogonal projection operator if and only if P is self adjoint and idempotent. 6

   b) Let X be a complex Banach algebra with identity e and G be the set of all invertible elements of X. Show that the mapping $g : G \to G$ defined by $g(x) = x^{-1}$ is continuous. 4

7. a) Give an example with proper justification of a bounded self adjoint linear operator on a Hilbert space which has no eigenvalue. 3

   b) State and prove Banach-Alaoglu theorem. 1 + 6

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

PG-Sc.-AP-17116

Special Paper : Applied Mathematics

Paper - XB(ii) : Mechanics of Solids

(Notations and symbols have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$
   a) What is meant by the line of shear stress ?
   b) Write down the difference between plane strain and plane stress problem.
   c) Based on which principle the variational method is applicable ?
   d) Write down the displacements in terms of deflections of thin elastic plate.
   e) Find the bending and twisting moments of a thin elastic plate.
   f) Define plane wave.
   g) Explain Tresca's criterion.
   h) Define stress deviator.

First Answer :

Second Answer :

Third Answer :

Fourth Answer :

Fifth Answer :

2. Deduce the Prandtl-Ruess stress-strain relations for plastic flow in an elasto-plastic medium in the form $\dot{s}_x = 2G\left(\dot{e}_x - \frac{\dot{W}}{2k^2}s_x\right)$ and two similar equations $\dot{\tau}_{yz} = G\left(\dot{\gamma}_{yz} - \frac{\dot{W}}{k^2}\tau_{yz}\right)$ and two similar equations. 10

3. Solve the problem of vibration of a thin rectangular plate with simply supported edge. 10

4. Find the displacement in a hemisphere if a constant force of magnitude P is applied to hemispherical surface along the direction of positive Z-axis. 10

5. What is Love waves ? Find the frequency equation for Love wave and show that it is dispersive in nature. 10

6. Show that in torsion problem, the torsion function ϕ satisfies the following Neumann problem $\nabla_1^2\varphi = 0$ in S and $\frac{\partial\varphi}{\partial n} = \frac{r\,dr}{ds}$ on L,
where the terms have their usual meaning. 10

7. a) State and prove the theorem of minimum potential energy. 6
   b) Explain how a plane problem of elasticity can be solved by using Airy's stress function. 4

First Answer :

Second Answer :

Third Answer :

Fourth Answer :