# NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 1A : Abstract Algebra**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

**Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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Q.P. Code : **PA/4/IA**

**PG-Sc.-AP-17097**

Signature of Evaluator with Date


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স্নাতকোত্তর পাঠক্রম **( P. G.)**


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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 1A : Abstract Algebra**

Name (in Block Letter) : .....

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Study Centre Name : ..... Code : .....

Q.P. Code : **PA/4/IA**

**PG-Sc.-AP-17097**


Received Answer Booklet
Signature with seal by the Study-Centre

## জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Let $G$ and $H$ be two groups and $f: G \to H$ be an epimorphism. If $G$ is cyclic, prove that $H$ is also cyclic.

b) Let $Z(G)$ denote the centre of a group $G$ and $N(a)$ denote the normalizer of $a$ in $G$. Prove that $a \in Z(G)$ iff $N(a) = G$.

c) Prove that a group of order 38 has a non-trivial normal subgroup.

d) Let $D$ be a Euclidean domain and $a, b, c \in D$. If $a \mid bc$ and $(a, b) = 1$, prove that $a \mid c$.

e) Consider the ring of integers $Z$. Let $p \in Z$ be a prime. Prove that ideal $< p >$ in $Z$ generated by $p$ is a maximal ideal of $Z$.

f) Let $F$ be a field consisting of $p^n$ elements where $p$ is a prime and $n$ is a positive integer. Prove that every element of $F$ is a root of the polynomial $x^{p^n} - x$ over $Z_p$.

g) Prove that the field extension $Q(\sqrt{2}, \sqrt{3})/Q$ is a simple extension where $Q$ is the field of rational numbers.

h) Let $f: R_1 \to R_2$ be a ring epimorphism. If $I$ is an ideal of $R_1$, prove that $f(I) = \{ f(i) : i \in I \}$ is an ideal of $R_2$.

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**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) Let $G_1$ and $G_2$ be two groups and $N_1$ and $N_2$ be normal subgroups of $G_1$ and $G_2$ respectively. If $G_1 \cong G_2$ and $N_1 \cong N_2$, is it true that the quotient groups $G_1/N_1$ and $G_2/N_2$ are isomorphic ? Justify your answer. 5

b) Let $H$ and $K$ be two normal subgroups of a group $G$ with $K \subseteq H$. Prove that $\frac{G/K}{H/K} \cong G/H$. 5

3. a) Let $G$ be a group and $G'$ be its derived subgroup. Prove that

i) $G'$ is a normal subgroup of $G$;

ii) If $H$ is a normal subgroup of $G$,

Prove that $G/H$ is commutative iff $G' \subseteq H$. 2 + 3

b) Let $G$ be a group. Prove that $Inn(G)$ is a normal subgroup of $Aut(G)$ where $Inn(G)$ is the set of all inner automorphisms of $G$ and $Aut(G)$ is the group of all automorphisms of $G$. 5

4. a) Let $G$ be a group of order $p^n$ where $p$ is a prime and $n$ is a positive integer. Prove that $Z(G) \neq \{e\}$ where $e$ is the identity element of $G$. 5

b) Let $G$ be a finite group. Prove that $|G| = \sum_a [G : N(a)]$ where the summation is over a complete set of distinct conjugacy class representatives. 5

5. a) Define prime ideal and maximal ideal in a commutative ring $R$ with identity. Prove that in a Boolean ring every prime ideal is a maximal ideal. 1 + 1 + 3

b) Let $R$ be a commutative ring with identity. Prove that every proper ideal $R$ is contained in a maximal ideal of $R$. 5

6. a) Prove that $Z[x]/_{<x>} \cong Z$ where $Z$ is the ring of integers and $<x>$ is the ideal of $Z[x]$ generated by $x$. 4

b) Let $R$ be a principal ideal domain and $p \in R$. Prove that $p$ is irreducible iff $p$ is prime. 6

7. a) Let $F/K$ be a finite field extension and $L$ be an intermediate field of $F/K$. Prove that $[F : K] = [F : L][L : K]$. 5

b) Let $f(x)$ be a non-constant polynomial of degree $n$ over a field $K$ and $S$ be a splitting field of $f(x)$ over $K$. Prove that $[S : K] \leq n!$. Is $S/K$ an algebraic extension ? Justify your answer. 5

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

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**MATHEMATICS**

**Paper - 1B : Linear Algebra**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

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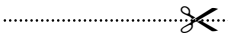
**PG-Sc.-AP-17098**

Signature of Evaluator with Date





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**MATHEMATICS**

**Paper - 1B : Linear Algebra**

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*( Unexplained Notations and Symbols have their usual meanings )*

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Let $T$ and $U$ be linear operators on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$. Describe $T$ & $U$ geometrically.

b) Let $T$ be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ and $U$ be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$. Prove that the linear transformation $UT$ is not invertible. Generalize this result.

c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator on $\mathbb{R}^2$ defined by $T(x_1, x_2) = (-x_2, x_1)$. Prove that for any real number $C$ the operator $(T - CI)$ is invertible.

d) Let $A = \begin{pmatrix} 2 & 1 & 0 & 0 & 3 \\ 0 & 2 & 2 & 0 & 4 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix} \in M_5(\mathbb{R})$.

Find the algebraic multiplicities and geometric multiplicities of the eigenvalues of $A$.

e) Let $a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$ be $\mathbb{R}^n$ be a unit vector. Find the minimal polynomial of the matrix $aa^T$ and hence discuss its diagonalizability.

f) Let $V$ be a vector space over a field $F$. Define (i) a symmetric bilinear form on $V$, (ii) a quadratic form on $V$.

g) Suppose $V$ and $W$ are two vector spaces over a field $F$ and $T : V \to W$ is a linear transformation. Prove that if $V$ is finite dimensional then rank $T$ is finite.

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**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) Let $V$ be a finite dimensional vector space and let $T$ be a linear operator on $V$ such that rank $T^2$ = rank $T$. Prove that the range and null space of $T$ have only the zero vector in common.

b) Suppose $T$ is a linear operator on a vector space $V$ such that $T^2 = T$ ( *i.e.,* $T$ is idempotent). Prove that

i) $v \in Im\ T$ if and only if $Tv = v$,

ii) $V = Im\,T \oplus ker\,T$

iii) If $V$ is finite dimensional then (i) and (ii) together implies that there exists an ordered basis $\mathfrak{B}$ of $V$ such that the matrix of $T$ with respect to $\mathfrak{B}$ is $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}_{n\times n}$ where $I_r$ is the $r \times r$ identity matrix and 0's are null matrices of suitable orders $(n = dim\ V)$. 4 + ( 2 + 2 + 2 )

3. a) Find a linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose image is spanned by $\{(1,0,-1),(2,1,3)\}$.

b) Does there exist a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,0) = (1,1)$ ? If yes, find such a linear transformation. Is it unique ? Answer with reasons.

c) Define an invariant subspace of a vector space $V$ with respect to a linear operator $T$ on $V$. Suppose $S$ and $T$ are two linear operators on a vector space $V$ such that $S \circ T = T \circ S$. Prove that $ker\ S$ and $Im\ S$ are invariant under $T$. 3 + 3 + 4

4. a) Let $A = \begin{pmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{pmatrix}$ where $a,b,c$ are positive real numbers satisfying $b^2 + c^2 < a < 1$. Prove that all the eigenvalues of $A$ are positive real numbers and hence conclude that the matrix is positive definite.

b) Let $V$ and $W$ be finite dimensional vector spaces over $R$. Let $T_1 : V \to V$ and $T_2 : W \to W$ be linear transformations whose minimal polynomials are respectively given by $f_1(x) = x^3 + x^2 + x + 1$ and $f_2(x) = x^4 - x^2 - 2$. Let $T : V \oplus W \to V \oplus W$ be the linear transformation defined by
$T(v,w) = (T_1(v), T_2(w))$
( *i.e.,* $T(v + w) = T_1(v) + T_2(w)$ for all $v \in V$, for all $w \in W$).
Find the minimal polynomial of $T$.
Is $T$ diagonalizable ? Answer with reason. 4 + 4 + 2

5. a) Let $A$ be a real square matrix. Prove that $A^*A$ is a positive semi-definite matrix. Also prove that if $A$ is invertible then $A^*A$ is positive definite.

b) Does there exist an inner product <, > on $\mathbb{R}^2$ such that $<(1,0),(0,1)> = -2$ ? If exists, find one such inner product. Is it unique ? Answer with reasons. 5 + 5

6. a) Let $A$ be the companion matrix of the polynomial $f(x) = x^4 - 5x^2 + 4$. Find $A$.
Let $T : M_4(\mathbb{R}) \to M_4(\mathbb{R})$ be the linear transformation defined by $T(B) = AB$ for all $B \in M_4(\mathbb{R})$ ($M_4(\mathbb{R})$ denotes the vector space of all 4 × 4 real matrices). Prove that $T$ is diagonalizable.

b) Find all possible Jordan forms and rational canonical forms of a matrix with characteristic polynomial $f(x) = (x+2)^5(x+3)^3$ and minimal polynomial $m(x) = (x+2)^2(x+3)^2$. ( 2 + 2 ) + ( 3 + 3 )

7. a) Let $V$ be a subspace of $\mathbb{R}[x]$ of real polynomials of degree at most 3. Equip $V$ with the inner product $< f, g > = \int_0^1 f(t)g(t)\mathrm{d}t$. Apply the Gram-Schmidt process to the basis $\{1, x, x^2, x^3\}$.

b) Let $W$ be the subspace of $\mathbb{R}^2$ spanned by the vector ( 3, 4 ). Using the standard inner product, let $E$ be the orthogonal projection of $\mathbb{R}^2$ onto $W$.
Find

i) a formula for $E(x_1, x_2)$.

ii) the matrix of $E$ in the standard ordered basis

iii) $W^{\perp}$ ( *i.e.*, $W$ perpendicular ). 4 + 6

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**MATHEMATICS**

**Paper - 2A : Real Analysis & Metric Spaces**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

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**PG-Sc.-AP-17099**

Signature of Evaluator with Date

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**MATHEMATICS**

**Paper - 2A : Real Analysis & Metric Spaces**

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

Q.P. Code : **PA/4/IIA**

**PG-Sc.-AP-17099**

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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Find the total variation of $f : [\,0,1\,] \to IR$ defined as

$$f(x) = \begin{cases} x\cos\frac{\pi}{x}, & \text{if } 0 < x \le 1 \\ 0, & \text{if } x = 0. \end{cases}$$

b) If $E$ is a bounded measurable set then show that there exists a subset $F \subseteq E$ such that $F$ is a countable union of closed sets with $m(F) = m(E)$.

c) Show that the set of rational numbers $Q$ is measurable.

d) Using definition of *R-S* integral, show that $\int_0^2 x^2 \mathrm{d}[x] = 5$; where $[x]$ denotes the largest integer $\le x$.

e) If $\{F_n\}$ is a sequence of sets of first category in a metric space $(X, \mathrm{d})$ then show that $\bigcup_{n=1}^{\infty} F_n$ is also a set of first category.

f) If $A$ and $B$ are two sets in a metric space ( $X$, d ) such that $A \cap B \neq \phi$, then show that $Diam(A \cup B) \le Diam(A) + Diam(B)$
where $Diam(S)$ stands for the diameter of $S$, for any $S \subseteq X$. What happens if the condition $A \cap B \neq \phi$ is not given ? Justify.

g) If $A$ and $B$ are disjoint compact sets in a metric space $(X, \mathrm{d})$ then show that there are disjoint open sets $G_1$, $G_2$ in $(X, \mathrm{d})$ such that $A \subseteq G_1$ and $B \subseteq G_2$.

h) Is the set $A = [2,3] \cup \left\{ 3 + \frac{1}{2^n} : n = 1,2,\ldots \right\}$ with usual metric induced from $IR$ a compact set ? Justify.

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) Let $\{E_k\}$ be an increasing sequence of measurable sets such that $\bigcup_{k=1}^{\infty} E_k = E$ is bounded. Show that $E$ is measurable and $m(E) = \lim_{n \to \infty} m(E_n)$. 4

b) If $f : [\, a, b\,] \to \mathbb{R}$ is a bounded measurable function and $J_1, J_2$ are open intervals in $\mathbb{R}$, then show that $f^{-1}(J_1 \cup J_2)$ is a measurable set in $[\, a, b\,]$. 3

c) Either prove or disprove :
Every measurable function is continuous but every continuous function need not be measurable. 3

3. a) If $f : [\, a,b\,] \to \mathbb{R}$ is summable and if $F(x) = \int_a^x f \, dt$ in $a \le x \le b$, then show that $F'(x) = f(x)$ almost everywhere in $[\, a, b\,]$. 5

b) If $f(x) = \frac{1}{2} + \cos x$ in $0 < x < 2\pi$, then find $f^+$ and $f^-$. 2

c) Find the Fourier series for $f : [\, -\pi, \pi\,] \to \mathbb{R}$
given by $f(x) = \begin{cases} x, & \text{if } -\pi < x < 0 \\ \pi - x, & \text{if } 0 < x < \pi \end{cases}$ 3

4. a) If $f : [-\pi, \pi] \to \mathbb{R}$ is a bounded and $R$-integrable and $f$ is periodic with period $= 2\pi$ and if $f$ is increasing in $(0, \alpha)$ where $0 < \alpha < a$, then show that
$\lim_{n \to \infty} \int_0^a f(t) \frac{\sin nt}{t} \, dt = \frac{\pi}{2} f(0+)$. 5

b) Show that Fourier series for $f(x) = x + x^2$ in $[-\pi, \pi]$ is
$$\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \left( \frac{\cos nx}{n^2} - \frac{\sin nx}{2n} \right).$$
What are the sums of the series at $x = \pm\pi$? Jusitify. 5

5. a) If two metrices $\rho$ and $d$ on a set $X (\neq \phi)$ are equivalent then show that a Cauchy sequence in $(X, \rho)$ is a Cauchy sequence in $(X, d)$ and conversely. 5

b) If $\sigma(x,y) = |\tan^{-1} x - \tan^{-1} y|$ for all $x, y \in \mathbb{R}$, then show that $(\mathbb{R}, \sigma)$ is a metric space. Is it complete ? Justify. 5

6. a) If a continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfies the property $f(x+y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$, then show that $f(x) = x f(1)$, for all $x \in \mathbb{R}$. 4

b) If $T$ is a contraction in a complete metric space $X$ and $x \in X$ show that $T\left( \lim_n T^n(x) \right) = \lim_n T^{n+1}(x)$. 4

c) Let $(X, d)$ be a metric space and $x_0 \in X$. Show that $\{ x \in X \mid 1 < d(x, x_0) < 5 \}$ is an open set in $(X, d)$. 2

7. a) Either prove or disprove :
   i) Continuous image of a connected space is connected.
   ii) Continuous image of a locally connected space is locally connected. 2 + 2

   b) Show that a continuous injective function of ( 0, 1 ) to real numbers with usual metric is a monotone function. 3

   c) If $A$ is a compact subset of a metric space ( $X$, $d$ ) and $x_0 \notin A$, then show that $d(x_0, A) = d(x_0, a_0)$ for some $a_0 \in A$. What happens if 'compactness' of $A$ is suppressed ? 3

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**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

# NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 2B : Complex Analysis**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

**Special credit will be given for precise and correct answer. Marks will be deducted for spelling mistakes, untidiness and illegible handwriting. The figures in the margin indicate full marks.**

Name (in Block Letter) : .....

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**PG-Sc.-AP-17100**

Signature of Evaluator with Date


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**MATHEMATICS**

**Paper - 2B : Complex Analysis**

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

Q.P. Code : **PA/4/IIB**

**PG-Sc.-AP-17100**


Received Answer Booklet
Signature with seal by the Study-Centre

## জরুরি নির্দেশ / Important Instruction

আগামী শিক্ষাবর্ষান্ত পরীক্ষায় **(T.E. Exam.)** নতুন ব্যবস্থা অর্থাৎ প্রশ্নসহ উত্তর পুস্তিকা **(QPAB)** প্রবর্তন করা হবে। এই নতুন ব্যবস্থার সঙ্গে পরীক্ষার্থীদের অভ্যস্ত করার জন্য বর্তমান অনুশীলন পত্রে নির্দেশ অনুযায়ী প্রতিটি প্রশ্নের উত্তর নির্দিষ্ট স্থানেই দিতে হবে।

**New system *i.e.* Question Paper Cum Answer Booklet (QPAB) will be introduced in the coming Term End Examination. To get the candidates acquainted with the new system, assignment answer is to be given in the specified space according to the instructions.**

**Detail schedule for submission of assignment for the PG Term End Examination June-2020/Dec.-2020**

| | | |
|---|---|---|
| 1. | Date of Publication | : 20/06/2020 |
| 2. | Last date of Submission of answer script by the student to the study centre | : 19/07/2020 |
| 3. | Last date of Submission of marks by the examiner to the study centre | : 16/08/2020 |
| 4. | Date of evaluated answer scripts distribution by the study centre to the students (Students are advised to check their assignment marks on the evaluated answer scripts and marks lists in the study centre notice board. If there is any mismatch / any other problems of marks obtained and marks in the list, the students should report to their study centre Co-ordinator on spot for correction. The study centre is advised to send the corrected marks, if any, to the COE office within five days. No changed / correction of assignment marks will be accepted after the said five days.) | : 23/08/2020 |
| 5. | Last date of submission of marks by the study centre to the Department of C.O.E. on or before | : 31/08/2020 |

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2 × 5 = 10

a) Show that $u(x,y)=\frac{1}{2}\log(x^2+y^2)$ is harmonic.

b) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}z^n$.

c) Examine the singularity of the function $f(z)=\sin\frac{1}{z}$ at $z=0$.

d) Find a bilinear transformation which have fixed points 0 and $\infty$.

e) Find where $f(z)=\sin z$ is conformal and find the critical points if any.

f) Find the residues of the function $f(z)=\frac{e^{iz}}{z^2+1}$ at its singularities.

g) Given that $f(z)$ is analytic in a region $R$ and $f'(z)=0$ in $R$. Show that $f(z)$ is constant in $R$.

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**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) State and prove Cauchy integral formula for derivative.

   b) If $C$ is a closed contour containing the origin inside it, prove that $\frac{a^n}{n!} = \frac{1}{2\pi i}\oint_C \frac{e^{az}}{z^{n+1}}dz$. 6 + 4

3. a) State and prove Liouville's theorem. From this deduce fundamental theorem of Classical Algebra. 4 + 2

   b) Prove that the function $f(z) = \frac{1}{z^2}$ is not uniformly continuous in $|z| \le 1$, but it is uniformly continuous in $\frac{1}{2} \le |z| \le 1$. 4

4. a) State and prove sufficient condition for analyticity of a complex valued function.

   b) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series valid for (i) $|z-1| > 1$, (ii) $0 < |z-2| < 1$. 6 + 4

5. a) State and prove Riemann's theorem on removable singularity for an analytic function.

   b) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n}$.

   c) If $f(z)$ has an isolated singularity at $z_0$ and $f(z) \to \infty$ as $z \to z_0$, then show that $f(z)$ has a pole at $z_0$. 5 + 3 + 2

6. a) State and prove Argument Principle.

   b) Show by the method of contour integration $\int_0^{\infty} \frac{\sin mx}{x}dx = \frac{\pi}{2}$. 5 + 5

7. a) Prove that a bilinear transformation transforms a circle into a circle and inverse points into inverse points.

   b) Show that the transformation $w = \frac{1-iz}{z-i}$ maps $|z| < 1$ into a circle in the $w$-plane whose centre is on the imaginary axis.

   c) Examine the singularities of the function $\sin\frac{1}{z-2} + \sin\frac{1}{z+2}$. 4 + 4 + 2

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

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স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 3A : Ordinary Differential Equations**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

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Q.P. Code : **PA/4/IIIA**

**PG-Sc.-AP-17101**

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**MATHEMATICS**

**Paper - 3A : Ordinary Differential Equations**

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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*Notations / symbols have their usual meanings.*

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Solve $\frac{dY}{dt} = AY$, where $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

b) In the following BVP, examine whether a Green's function exists and if it does then construct it :
$y'' + y' = 0$, $y(0) = y(1)$, $y'(0) = y'(1)$.

c) Find the regular singular point for the equation $t^2y'' + ty' + (t^2 - 4)y = 0$. Is $t = 2020$ a regular singular point for the above equation ?

d) Compute the solution of $y''' + y'' + y' + y = 2020$ with the initial conditions $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

e) Using the Picard's method of successive approximation, find the third approximation of the solution of the equatoin : $y' = t + y^2$, $y(0) = 0$.

f) Determine the nature of the critical points for the system $\dot{x} = 4x - 3y$, $\dot{y} = 8x - 6y$, where $\dot{x} = \frac{dx}{dt}$ etc.

g) Show that $L_n''(0) = \frac{n}{2}(n-1)$.

h) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) Find the general solution of the equation $(t^2+1)y'' - 2ty' + 2y = 6(t^2+1)^2$, given that $y = t$ and $y = t^2 - 1$ are linearly independent solutions of the corresponding homogeneous equation. 5

b) If $R$ is either a rectangle $|x - x_0| \le h$, $|y - y_0| \le k$ or a strip $|x - x_0| \le h, |y| < \infty$ and if $f(x,y)$ is a real-valued function defined on $R$ such that $\frac{\partial f}{\partial y}$ exists, is continuous on $R$ and $\left|\frac{\partial f}{\partial y}\right| \le \overline{K}\ \forall (x,y) \in R$ for a positive constant $\overline{K}$, then prove that $f(x,y)$ satisfies a Lipschitz condition on $R$ with Lipschitz constant $\overline{K}$. Is the converse true ? Justify your answer. 5

3. a) Let the vector functions $\bar{\phi}_1, \bar{\phi}_2, \ldots \bar{\phi}_n$ be $n$ solutions of the homogeneous linear vector differential equation $\frac{dY}{dt} = AY$ on the interval $a \le t \le b$. Then prove that either

$W(\bar{\phi}_1, \bar{\phi}_2, \ldots, \bar{\phi}_n) = 0\ \forall\ t \in [a,b]$ or $W(\bar{\phi}_1, \bar{\phi}_2, \ldots, \bar{\phi}_n) \ne 0\ \forall\ t \in [a,b]$,

where $W$ is the Wronskian. 5

b) Find the nature and the stability of the critical points of the system $\frac{dx}{dt} = -ax + y$, $\frac{dy}{dt} = -x - ay$ for $a < 0$ and $a > 0$. 5

4. a) Find the Green's function and hence solve the following BVP :

$y'' + k^2 y = f(x)$ $(k \ne \pi)$,

$y(0) = \alpha$, $y'(1) = \beta$,

where $\alpha, \beta$ are some real constants. 5

b) Show that the general solution of the equation $y'' + 3y' + 2y = g(t)$ is bounded on $[0,\infty)$ if $g(t)$ is bounded for all $t$ in $[0,\infty)$. 5

5. a) Solve the following system of ODEs :

$$\frac{dY}{dt} = AY,\ A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{bmatrix},\ Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$ 5

b) Find the eigenvalues and eigenfunctions for the following S-L equations :

$y'' + \lambda y = 0$, $-\pi \le t \le \pi$

subject to

$y(-\pi) = y(\pi)$,

$y'(-\pi) = y'(\pi)$. 5

6. a) Prove that $J_n(x) = \frac{1}{2}\left[J'_{n-1}(x) - J_{n+1}(x)\right]$. 5

b) Find the power series soluton of the differential equation $y'' + ty' + t^2 y = 0$ in powers of $x$ about $x = 0$. 5

7. a) Prove that $\int_0^\infty e^{-rx} L_n(x)\,dx = \frac{1}{r}\left(1-\frac{1}{r}\right)^n$ , $r \neq 0$. 5

b) Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$. 5

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**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

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স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 3B : Partial Differential Equations and Special Function**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

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Q.P. Code : **PA/4/IIIB**

**PG-Sc.-AP-17102**

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**MATHEMATICS**

**Paper - 3B : Partial Differential Equations and Special Function**

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

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**PG-Sc.-AP-17102**


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## জরুরি নির্দেশ / Important Instruction

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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Symbols / Notations have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) Form a partial differential equation by eliminating the arbitrary constants $a$ and $b$ from $z = ae^{-b^2 t}\cos px$.

b) Construct a partial differential equation by eliminating the arbitrary function $f$ from $z = f\left(xy/z\right)$.

c) Solve : $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

d) Obtain the complementary function of $(D^3 - 7DD'^2 - 6D'^3)z = e^{2x+y}$.

e) Classify the partial differential equation $u_{xx} + xu_{yy} = 0$, $x \neq 0$.

f) If $\varphi$ is a harmonic function in $\mathbb{R}$ and $\frac{\partial \varphi}{\partial n} = 0$ on $\partial\mathbb{R}$, then $\varphi$ is a constant in $\overline{\mathbb{R}}$.

g) Find the adjoint to the operator $L$, defined by

$L(z) = AZ_{xx} + BZ_{xy} + CZ_{yy} + DZ_x + EZ_y + F_z$,

where $A, B, C, D, E, F$ are functions of $x$ and $y$ only.

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. a) Form a partial differential equation by eliminating the arbitrary function $f$ from $xyz = f(x+y+z)$. 4

b) Solve : $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2z(x^2 + y^2)$. 4

c) Solve : $(D^2 + DD')z = \sin(x+y)$. 2

3. a) Find the integral surface of the equation

$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$

through the curve $xz = a^3$, $y = 0$. 4

b) Solve : $px + qy = pq$ by Charpit's method. 4

c) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $u = u_0 \sin\frac{2\pi x}{L}$, $0 \le x \le L$, and then released. Find the displacement of any point $x$ of the string at any time $t > 0$. 2

4. Verify that the Green's function for the equation

$$\frac{\partial^2 u}{\partial x \partial y} + \frac{2}{x+y}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$$

subject to $u = 0$, $\frac{\partial u}{\partial x} = 3x^2$ on $y = x$, is given by

$$u(x,y;\xi,\eta) = \frac{(x+y)\{2xy + (\xi-\eta)(x-y) + 2\xi\eta\}}{(\xi+\eta)^3}$$

and obtain the solution of the equation in the form

$u = (x-y)(2x^2 - xy + 2y^2)$. 10

5. a) Reduce the partial differential equation

$x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$, $x > 0$

into canonical form and hence solve it. 7

b) If the Dirichlet problem for a bounded region has a solution, then it is unique. 3

6. If $u(x,t)$ be a continuous function which is a solution of $u_t = u_{xx}$ in the rectangle $R$ : $0 \le x \le L$, $0 \le t \le T$, then prove that the maximum value of $u$ is attained either on the boundary $t = 0$ on the boundaries $x = 0$ and $x = L$.

Hence show that the solution of the following boundary value problem for $u(x,t)$ is unique :

$u_t = u_{xx}$, $0 \le x \le L$, $0 \le t \le T$,

$u(0,t) = f(t)$, $u(L,t) = g(t)$, $u(x,0) = \varphi(x)$.

where $f(t)$, $g(t)$, $\varphi(x)$ are continuous functions and $\varphi(0) = f(0)$, $\phi(L) = g(L)$. 10

7. Solve the following problem for $u(x,t)$ :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ 0 \le x \le L,\ t \ge 0$$

$$u(0,t) = u(L,t) = 0,\ t \ge 0$$

$$u(x,0) = \begin{cases} \frac{x}{b}, & 0 \le x \le b \\ \frac{L-x}{L-b}, & b \le x \le \infty \end{cases}.$$ 10

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

# NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 4A : Numerical Analysis**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

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Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

| To be filled by the Candidate | Serial No. of question answered | | | | | | | | | | | | | | **TOTAL** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| For Evaluator's only | Marks awarded | | | | | | | | | | | | | | |

Q.P. Code : **PA/4/IVA**

**PG-Sc.-AP-17103**

Signature of Evaluator with Date


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স্নাতকোত্তর পাঠক্রম **( P. G.)**


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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 4A : Numerical Analysis**

Name (in Block Letter) : .....

Enrolment No. | | | | | | | | | | | | |

Study Centre Name : ..... Code : .....

Q.P. Code : **PA/4/IVA**

**PG-Sc.-AP-17103**


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## জরুরি নির্দেশ / Important Instruction

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2 × 5 = 10

   a) Define degree of precision of a mechanical quadrature formula.

   b) What are ill-conditioned systems ? Give suitable examples.

   c) Find the condition number of $f(x)=\sqrt{x+1}-\sqrt{x}$ and test whether the function is ill-conditioned or not.

   d) If $T_n(x)=\cos(n\cos^{-1}x)$ represents $n$th degree Chebyshev polynomial, then show that $T_{n+1}=2xT_n(x)-T_{n-1}(x)$, $(n\geq 1)$.

   e) Is $y_{n+3}=y_{n+1}+2hf(x_n,y_n)$ a multistep method ? Justify.

   f) Show geometrically that when $|\phi'(x)|>1$ near the root $r=\alpha$, the iteration process $x_n=\phi(x_{n-1})$ diverges.

   g) How is the least eigen pair of a non-singular matrix $A$ determined by the power method ?

   h) Use appropriate formula for computing roots of the following equation :

   $x^2-100\cdot 001x+1=0$

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

**Fifth Answer :**

2. Describe a finite difference scheme for finding numerical solution of a second order ordinary differential equation with prescribed boundary conditions. 10

3. Describe briefly Bairstow's method for finding quadratic factor of a real polynomial of degree $n(\geq 3)$. 10

4. State Gauss Quadrature problem and obtain the Gauss-Legendre solution of it.

5. Obtain an explicit finite difference scheme for solving the parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

$$u(x,0) = f(x),\ 0 \leq x \leq 1$$

$$u(0,t) = 0 = u(1,t),\ t > 0\ .$$

Under what condition the scheme is absolutely stable ? 10

6. Describe cubic spline. Describe briefly the method of construction of cubic spline function. What are end point conditions of natural cubic spline ? 10

7. Given a set of $(n+1)$ points $(x_i, y_i)$ of two variables $x$ and $y$ ( $i$ = 0, 1, 2, ..., $n$ ). Obtain the least-square straight line $y = a + bx$ to fit into the given data. Apply this method to the following set of points ( 0, 1·0), (1, 2·9), (2, 4·8), (3, 6·7), (4, 8·6) and obtain the least-square straight line. 10

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**

# NETAJI SUBHAS OPEN UNIVERSITY

স্নাতকোত্তর পাঠক্রম **( P. G.)**

অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 4B : Computer Programming & Its Application To Numerical Analysis**

পূর্ণমান : ৫০ **QUESTION PAPER CUM ANSWER BOOKLET** মানের গুরুত্ব : ২০%

(Full Marks : 50) (Weightage of Marks : 20%)

**পরিমিত ও যথাযথ উত্তরের জন্য বিশেষ মূল্য দেওয়া হবে। অশুদ্ধ বানান, অপরিচ্ছন্নতা এবং অপরিষ্কার হস্তাক্ষরের ক্ষেত্রে নম্বর কেটে নেওয়া হবে। উপান্তে প্রশ্নের মূল্যমান সূচিত আছে।**

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Q.P. Code : **PA/4/IVB**

**PG-Sc.-AP-17104**

Signature of Evaluator with Date


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অনুশীলন পত্র **(Assignment) :** জুন, ২০২০/ ডিসেম্বর, ২০২০ **(June-2020/Dec.-2020)**

**MATHEMATICS**

**Paper - 4B : Computer Programming & Its Application To Numerical Analysis**

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Enrolment No. | | | | | | | | | | | | |

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# এখানে কিছু লিখবেন না

# Do Not Write Anything Here

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Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

a) What are the differences between `"a"` and `'a'` in C language ?

b) Which one of the following statements is not equivalent to the other two (assuming that loop bodies are the same) ?

i) `for (i=0; i<10; i++) {body}`

ii) `for (i=0; i<10; ++i) {body}`

iii) `for (i=0; i++<10;) {body}.`

c) What is the output of the program segment ?

```
int i=3, j=4, k=5;
printf ("%d\n", i<j || ++j<k);
printf ("%d%d%d", i,j,k);
```

d) How are 'stack' and 'queue' different ?

e) Find out the errors (if any) :

i) `if(x ≤ y) max == y;`

ii) `for( ; i>0; --i);`

f) What is meant by the address of memory cell ? How are addresses usually numbered ?

g) What output does the following for statement produce ?

```
for (i=10; i>=1; i/=2)
   printf ("%d", i++);
```

h) Explain '%f' format specification.

---

**First Answer :**

**Second Answer :**

**Third Answer :**

---

**Fourth Answer :**

**Fifth Answer :**

2. a) Write a C program to display the following output : 5

```
*
* *
* * *
* * * *
* * * * *
```

b) Explain 'switch' statement in C. 2

c) Explain why the output of the following 'for' statement produces an infinite loop. 3

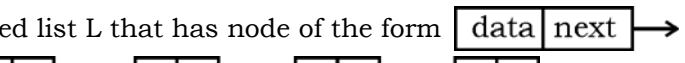
```
for (i=10; i>1; i=i/2)
     printf ("%d\n", i++);
```

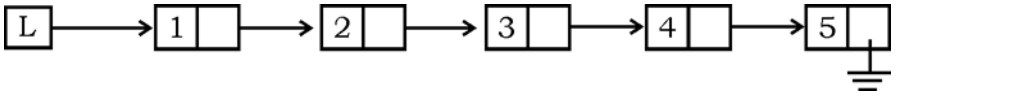
3. a) Rewrite the following function to use pointer arithmetic instead of array subscripting (In other words, eliminate the variable *i* and all uses of the [ ] operator.). Make as few changes as possible. 3

```
void store-zeros (int a[ ] , int n)
{
     int i ;
     for ( i=0; i<n; i++ )
           a [i] =0;
}
```

b) Suppose that high, low and middle are all pointer variables of the same type, and that low and high point to elements of an array. Why is the following statement illegal, and how could it be fixed ? 1 + 2

```
middle = ( low + high )/2;
```

c) Consider the following linked list L that has node of the form 


What does the following function print if we pass the linked list as input argument ? **4**

```
void func (struct node *L)
{
   if (L ! = NULL)
      {
          func(L → next);
          printf("%d",L → data);
       }
}
```

4. a) Find below a program in C. Determine the output after executing the program.

```
void main ( )
{
    int a = 5,b,*p,*q;
        p = &a;
        b = *p/2+10;
        q = p;
    printf ("a=%d,b=%d,*p=%d,*q=%d",a,b,*p,*q);
}
```

Also explain how the arithmetic expression **`*p/2+10`** is processed. 4

b) Translate each 'infix' expression into corresponding 'postfix' expression.

i) $A + B * C - (D + E) * C$

ii) $(A + B \uparrow D)/(E - F) + G$ 2 + 2

c) What are P and NPC problems ? 2

5. a) Suppose two linked list $L_1$ and $L_2$ are given. Write a procedure to concatenate these two list. 4

b) Find below the algorithmic steps of 'Babylonian Method' for finding square root of a positive real number *S*.

i) Guess any positive number $X_0$.

ii) Apply the formula $X_1 = (X_0 + S / X_0)/2$. The no. $X$ is better approximation of $\sqrt{S}$.

iii) Apply the formula $X_{n+1} = (X_n + S / X_n)/2$ until $| X_{n+1} - X_n |$ becomes smaller than 0·001.

Write a recursive function in C for above method and find out the value of $\sqrt{8}$ using this. 4 + 2

6. a) The parenthesis must appear in balanced fashion in any valid arithmetic expression. Balanced parenthesis means that each opening symbol has a corresponding closing symbol and the pairs of parenthesis are properly nested. Consider following two arithmetic expressions :

i) $[(x + y) * \{(a - b) * c\} - d] \rightarrow$ Balanced

ii) $(x + y) * [\{(a - b) + c) \rightarrow$ Not balanced.

Write a program in C for balanced parenthesis problem. Assume that '(', '{', '[' are opening parenthesis and ')', '}', ']' are closing parenthesis. You can use stack data structure. 6

b) Explain 'break' and 'continue' statements in C with an example. 2 + 2

7. a) Let 'DOUBLE' be the following macro :

```
# define DOUBLE(x)2*x
```

Find out the value of DOUBLE (1+2). Justify why the result is not 6. Rectify the macro definition so that DOUBLE (1+2) prints 6. 1 + 2 + 1

b) Explain following statements in C with an example :

i) nested if

ii) multi-alternative if. 2 + 2

c) Find out suitable values of $c_1, c_2$ and $n_0$ such that $2n + 5 = \theta(n)$. ($c_1, c_2$ and $n_0$ have their usual meanings as in the definition of $\theta(n)$ ). 2

---

**First Answer :**

**Second Answer :**

**Third Answer :**

**Fourth Answer :**